Topic I $\Rightarrow$ { Thermal Equilibrium Plasma

$\Rightarrow$ Thermal Equilibrium Plasma : **Basic Ideas**

- simplest possible dynamics question

$\Rightarrow$ What is spectrum of thermal equilibrium fluctuations in plasma?

- answer $\Rightarrow$ determined by balance between

$\rightarrow$ emission and absorption

$\rightarrow$ Fluctuation $\leftrightarrow$ Dissipation

What is key physics of each?

} $\Rightarrow$ Physics of F.D.T.

Generic Consideration

Consider some simple examples:

simplest

- particle undergoing Brownian force on fluid

$\rightarrow$ see 6a.)

particle in fluid at temp T (e.g. ...)

$$m\frac{d\underline{v}}{dt} = -\gamma m\underline{v} + \underline{f}$$

$-\gamma m \underline{v}$: Stokes drag $\sim v$

$\underline{f}$ $\rightarrow$ thermal fluctuations

$\tilde{f}$ $\rightarrow$ random (statistical) so uncorrelated in time

$$\langle \tilde{f}(t_1)\,\tilde{f}(t_2)\rangle = 2\tilde{f}_0^2\,\tau_c\,\delta(t_1-t_2)$$

τ_c $\rightarrow$ correlation time

auto-correlation function.

Note:

- standard notation for Stokes drag is:

$\mapsto$ mass of Brownian particle

$$m \frac{d\underline{v}}{dt} = -\gamma \underline{v} + \underline{\tilde{f}}$$

$$\gamma = 6\pi \eta a$$

$$\eta = \rho \nu$$

$\hookrightarrow$ fluid mass density

so, in these notes:

$\gamma \rightarrow \gamma/m$, as write;

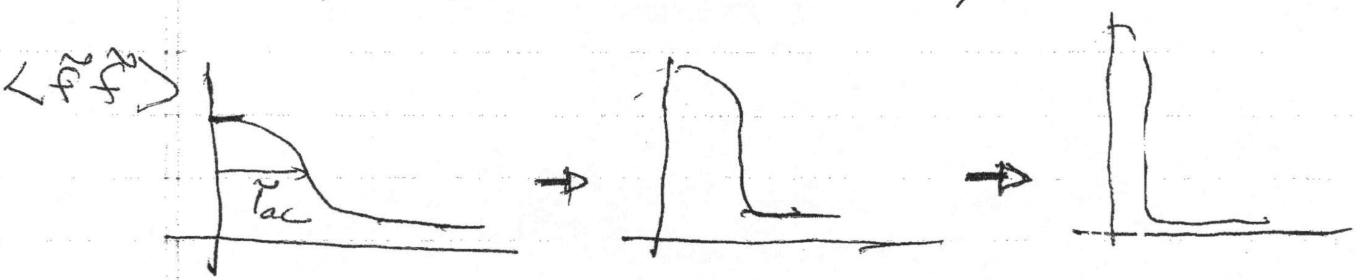
$$\left[m \frac{d\underline{v}}{dt} = -m(\gamma/m)\underline{v} + \underline{\tilde{f}} \right.$$

What is τ_c ? → spectral auto-correlation time
(self-coherence)

→ measures self-correlation of $\tilde{\ }$ random force.

i.e. if stationary,

$$\langle \tilde{f}(0)\, \tilde{f}(\tau) \rangle = \langle \tilde{f}(t_1)\, \tilde{f}(t_2) \rangle$$



⇒ for "white noise"
$\tau \ll$ all other time scales

now,

$$\frac{d\tilde{v}}{dt} + \gamma \tilde{v} = \frac{\tilde{f}}{m}$$

$$\tilde{v}(t) = e^{-\gamma t}\tilde{v}(0) + \int_0^t dt'\, e^{-\gamma(t-t')} \frac{\tilde{f}}{m}(t')$$

{ no correlation i.e. with $\tilde{f}$

cross terms

Now $$|\tilde{v}|^2 = e^{-2\gamma t}|\tilde{v}(0)|^2 + \langle \cancel{\tilde{v}\tilde{f}} \rangle$$
$$+ \int_0^t dt'\, e^{-\gamma(t-t')} \frac{\tilde{f}}{m}(t') \int_0^t dt''\, e^{-\gamma(t-t'')} \frac{\tilde{f}}{m}(t'')$$

$$|\tilde{v}|^2 = e^{-2\gamma t}|\tilde{v}(0)|^2 + \int_0^t dt' \int_0^t dt'' e^{-\gamma(t-t')} e^{-\gamma(t-t'')} \frac{\tilde{f}^*(t')\tilde{f}(t'')}{m^2}$$

$$\langle |\tilde{v}|^2 \rangle = e^{-2\gamma t}\langle |\tilde{v}(0)|^2 \rangle + \int_0^t dt' \int_0^t dt'' e^{-\gamma(t-t')} e^{-\gamma(t-t'')} \frac{\langle \tilde{f}(t')\tilde{f}(t'')\rangle}{m^2}$$

$\langle\ \rangle$ ensemble, statistical average

(2 for symmetrization)

$$= e^{-2\gamma t}\langle |\tilde{v}(0)|^2 \rangle + \int_0^t dt' \int_0^t dt'' e^{-\gamma(t-t'')} e^{-\gamma(t-t')} \frac{2\tilde{f}_0^2\tau_c\,\delta(t'-t'')}{m^2}$$

$$= e^{-2\gamma t}\langle |\tilde{v}(0)|^2 \rangle + \int_0^t dt' e^{-2\gamma(t-t')} \frac{2\tilde{f}_0^2\tau_c}{m^2}$$

$$= e^{-2\gamma t}\langle |\tilde{v}(0)|^2 \rangle + e^{-2\gamma t} \frac{2\tilde{f}_0^2\tau_c}{m^2} \frac{1}{2\gamma}\left(e^{2\gamma t}-1\right)$$

$$= e^{-2\gamma t}\langle |\tilde{v}(0)|^2 \rangle + \frac{\cancel{2}\tilde{f}_0^2\tau_c}{\cancel{2}\gamma m^2}\left(1-e^{-2\gamma t}\right)$$

so for t large ($\gamma t \gg 1$)

$$\boxed{\langle \tilde{V}^2 \rangle \cong \frac{\tilde{f}_0^2 \tau_c}{\gamma m^2}}$$

but $m \frac{\langle \tilde{V}^2 \rangle}{2} \equiv T$ → bath of T!

$$\Rightarrow \quad T \cong \frac{\tilde{f}_0^2 \tau_c}{2 \gamma m}$$

simple

$$\boxed{\frac{\tilde{f}_0^2 \tau_c}{m^2} = \gamma \frac{T}{m}}$$

→ Fluctuation - dissipation theorem

i.e. → given
- noise $(\tilde{f}_0^2 \tau_c)$
- damping (γ)
- temperature (T)

must have:

$$\boxed{(\text{noise}) = (\text{damping})\, T}$$

→ given 2 of 3 ⇒ deduce third!

$$\frac{\partial}{\partial t}\tilde{v} + \gamma \tilde{v} = \frac{\tilde{f}}{m}$$

$\tilde{v}\,\times$

stationarity

$$\frac{\partial}{\partial t}\left\langle \frac{\tilde{v}^2}{2}\right\rangle + \gamma\langle \tilde{v}^2\rangle = \left\langle \frac{\tilde{f}\tilde{v}}{m}\right\rangle$$

i.c.

but
$$\tilde{v}(t) = e^{-\gamma t}\tilde{v}(0) + \int_0^t dt'\, e^{-\gamma(t-t')}\frac{\tilde{f}(t')}{m}$$

$$\langle \tilde{v}^2\rangle = \frac{T}{m} = \frac{1}{\gamma}\left\langle \tilde{f}\int_0^t dt'\, e^{-\gamma(t-t')}\frac{\tilde{f}(t')}{m^2}\right\rangle$$

$$\langle \tilde{f}(t)\tilde{f}(t')\rangle = |\tilde{f}|^2 \tau_c\, \delta(t-t')$$

$$\langle \tilde{v}^2\rangle = \frac{T}{m} = \left(\frac{1}{\gamma}\right)\frac{|\tilde{f}|^2\tau_c}{m^2}$$

$$\boxed{\left\langle \frac{\tilde{f}^2}{m}\right\rangle \tau_c = \gamma T}$$

$$\frac{\tilde{f}_0^{\,2}\, \tau_0}{m} = \gamma T$$

but $m\gamma \to \gamma'$ (usual)

$\therefore$

$$\frac{\tilde{f}_0^{\,2}\, \tau_0}{m} = \frac{\gamma'}{m}\, T$$

$$\boxed{f_0^2 \tau_0 = \gamma' T}$$ $\leadsto$ standard form.

→ equilibrium:

→ emission by noise

→ absorption by damping

⇒ balance matches T!

$T \oplus$ damping → noise

note: alternatively

$$(-i\omega + \gamma)\tilde{V}_\omega = \tilde{f}_\omega / m$$

$$|\tilde{V}_\omega|^2 = \frac{|\frac{\tilde{f}_\omega}{m}|^2}{(\omega^2+\gamma^2)}$$

white noise: spectral intensity flat

$$\int d\omega\, |\tilde{V}_\omega|^2 = \frac{2T}{m} = \frac{|\tilde{f}|^2}{m^2}\int \frac{d\omega}{\omega^2+\gamma^2}$$

$$= \frac{|\tilde{f}_\omega|^2/m^2}{\gamma}$$

$$\frac{|\tilde{f}_\omega|^2}{m^2} = 2\gamma \frac{T}{m}$$

→ same

→ factors ⇔ normalization

note

$$\underbrace{|\tilde{V}_\omega|^2}_{\text{response spectral density}} = \frac{\overbrace{|\tilde{f}_\omega|^2/m^2}^{\text{noise spectral density}}}{\omega^2+\underbrace{\gamma^2}_{\text{damping}}}$$

$$= \frac{|\tilde{f}_\omega|^2/m^2}{\underbrace{|r(\omega)|^2}_{\text{response function [damping ⇔ width]}}}$$

i.e. $\frac{T}{m} = \int \frac{|\tilde{a}|^2}{|r(\omega)|^2} d\omega$

of oscillator:

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{\tilde{f}}{m}$$

$$|\tilde{x}_\omega|^2 = \frac{|\tilde{f}_\omega|^2/m^2}{(\omega_0^2-\omega^2)^2+(\gamma\omega)^2}$$

note: $$T/m = \int \frac{|\tilde{a}|^2}{|r_r(\omega)|^2 + |r_{IM}(\omega)|^2} \, d\omega$$

if → $|\tilde{a}(\omega)|^2$ broad

→ $r(\omega)$ has lines, so

$$r_r(\omega) = (\omega - \omega_0) \frac{\partial r}{\partial \omega}$$

$$T/M = \int \frac{|\tilde{a}|^2 \, d\omega}{(\omega-\omega_0)^2 \left(\frac{\partial r}{\partial \omega}\right)^2 + |r_{IM}(\omega_0)|^2}$$

$$= |\tilde{a}(\omega_0)|^2 \int \frac{d\omega / |r_{IM}(\omega_0)|^2}{\left[\frac{(\omega-\omega_0)^2}{|r_{IM}(\omega_0)|^2} \left|\frac{\partial r_n}{\partial \omega}\right|^2 + 1\right]}$$

$$\cong \frac{|\tilde{a}(\omega_0)|^2}{|r_{IM}(\omega_0)| \left|\frac{\partial r_n}{\partial \omega}\right|_{\omega_0}}$$

$$|\tilde{a}(\omega_0)|^2 = \left(r_{IM} \, T/m \right)\Big|_{\omega_0} \left|\frac{\partial r_n}{\partial \omega}\right|_{\omega_0}$$

noise dissipn Temp

→ Fluctuations set by

- noise
- damping
- collective modes

response

i.e. $\omega \simeq \omega_0$

natural frequency

→ $2\left(\frac{1}{2} k x^2\right) = 2\left(\frac{m\omega_0^2}{2} x^2\right) = T$

sets condition

Lesson: → Thermal equilibrium spectrum set by

- collective modes
- damping

resonances

- noise

→ F-D Thm links these, explicitly

→ For plasma, thermal equilibrium requires understanding

→ noise
→ collective modes
→ damping